Ex 1 Does $\displaystyle\sum_{n=0}^{\infty} \frac{2^n}{n!}$ converge or diverge?

(a) Converges

(b) Diverges

Q. Why can't we use the integral test to determine the behavior of the above series?

Q. How do we work with factorials?

Q. Consider the sequence defined by $a_n = r^n$ for $n \geqslant 0$ and the corresponding series $\sum_{n=0}^{\infty} a_n$.

(a) Under what conditions does the series converge?

(b) What is the ratio $\frac{a_{n+1}}{a_n}$?

(c) Rewrite the conditions from (a) to make a statement like "$\sum_{n=0}^{\infty} a_n$ converges if and only if [insert-something-about-$\frac{a_{n+1}}{a_n}$-here]"

Q. What is $\frac{(n+1)!}{n!}$?

Thm: (The Ratio Test) Suppose $\sum_{n=1}^{\infty} a_n$ is a series with nonzero terms and suppose _____.

Then

(a) If _____, then $\sum_{n=1}^{\infty} a_n$ _____

(b) If _____, then $\sum_{n=1}^{\infty} a_n$ _____.

(c) If _____, then the test _____.

Ex 2 Determine the behavior of $\sum_{n=1}^{\infty} \frac{(n!)^3}{(3n)!}$.

Ex 3 Use the ratio test to determine the behavior of $\sum_{n=1}^{\infty} \frac{2n^3 + 7n - 50}{(-2)^n}$.

(a) Converges

(b) Diverges

Q. For what types of series is the ratio test helpful?

Ex 4 What happens if we try to use the ratio test on p-series?

Ex 5 Does $\sum_{n=1}^{\infty} \frac{n^{3n}}{(2n+7)^n}$ converge or diverge?

(a) Converges

(b) Diverges

Q. Consider the sequence defined by $a_n = r^n$ for $n \geqslant 0$ and the corresponding series $\sum_{n=0}^{\infty} a_n$.

(a) Under what conditions does the series converge?

(b) What is the root $\sqrt[n]{a_n} = |a_n|^{1/n}$? (and why is the absolute value necessary?)

(c) Rewrite the conditions from (a) to make a statement like "$\sum_{n=0}^{\infty} a_n$ converges if and only if [insert-something-about-$|a_n|^{1/n}$-here]"

Thm: (The Root Test) Suppose $\sum_{n=1}^{\infty} a_n$ is a series with _____. Then

(a) If _____, then $\sum_{n=1}^{\infty} a_n$ _____.

(b) If _____, then $\sum_{n=1}^{\infty} a_n$ _____.

(c) If _____, then the test is inconclusive.

Ex 6 Determine the behavior of $\sum_{n=1}^{\infty} \frac{n^{3n}}{(2n+7)^n}$.

Ex 7 Determine the behavior of $\sum_{n=1}^{\infty} \left(\frac{n}{n+1}\right)^{n^2}$.

(a) Converges

(b) Diverges

Ex 8 Determine the behavior of $\sum_{n=2}^{\infty} (\ln(n))^n$.

(a) Converges

(b) Diverges

Q. For what types of series is the root test helpful?